EXTERNAL EXAMINATION SEM 6TH NON-CBCS (C.No. MA 601)

Time Allowed- 3 Hrs SUBJECT: MATH Maximum Marks- 80

(Each question carry 20 marks, Do any 4)

1.(a) Define vector space. Prove that union of two subspaces is a subspace iff one of them is a subset of other.

(b) Define smallest subspace, if S and T are any subsets of a vector space V(F),prove that

i) $S \subset L(T) \Rightarrow L(S) \subset L(T)$ ii) $S \subset T \Rightarrow L(S) \subset L(T)$ iii) $L(S \cup T = L(S) + L(T)$

Q2.(a) State and prove Existence theorem .

(b) Show that the set $B = \{ 1 , x , x^2 , \dots , x^m \}$ of (m+1) polynomial is a basis set of the vector space

$P_m(R)$ of all polynomials of degree m over $\mathbb{R}$.

Q3. (a) Define countabe set, uncountable set. Prove that cartesian product of two countable set is a countable.

(b) Prove that union and intersection of two closed set is a closed set.

Q4. (a) Let (X , d) be any metric space. Then show that the metric defind on X are bounded i) $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$

ii) $d_2(x, y) = \min \{ 1, d(x, y) \}$.

(b) Define interior, exterior point. Prove that every covergent sequence in a metric space is a cauchy sequence.

Q5. (a) Let V be a vector space over a field F. Then prove that i) Every super set of a L.D. set of vectors is L.D. ii) Any subset of a L.I. set of vectors is L.I.

(b) Define Isomorphic vector space. Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

Q6. (a) Define Linear transformation. Show that T: $V_3 \mathbb{R} \longrightarrow V_2 \mathbb{R}$ defined by T(x, y, z) = (x – y + z, 2x) is a linear transformation.

(b) If V(F) and W(F) are vector spaces and T: V → W is a linear transformation. Suppose V is of dimension n. Prove that Rank T + Nullity T = dim V.

Q7. (a) Let T be a linear transformation defined by T(x, y, z) = (x + y, y + z, x + z). Then find the matrix representation of T relative to the usual basis.

(b) Define dual basis,find the dual basis of { (1, -2 , 3), (1, -1 , 1) , (2, -4 , 7) } be given basis of $\mathbb{R}^3$.

Q8.(a) Define quotient space . Show that dim $(\frac{V}{W}) = \dim(v) - \dim(W)$.

(b)Find the value of k so that the vectors $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} and \begin{bmatrix} k \\ 0 \\ 1 \end{bmatrix}$ are L.D.